

AC ELECTRICAL FUNDAMENTALS TRIAL MID-SEMESTER TEST

- This test is **closed** book, calculator permitted.
- Answer questions in the spaces provided.
- Clearly label all currents, resistors and voltage drops in the circuits and state any assumptions in order to obtain a full mark
- When calculating values, show clearly all steps, starting with the formula, then substituting with numbers and finally show the measuring units of the obtained result. Otherwise **NO MARKS** are given
- It is permitted to use the formula sheets, given at the back of your lab book.
- Time permitted 1½ hours.
- **60 MARKS TOTAL (70% pass)**

Q1) In a certain magnetic field the cross-sectional area is 50 cm² and the flux is 1500 μWb.
What is the flux density?

[2 marks]

$$B = \frac{\Phi}{A} = \frac{1500 \times 10^{-6}}{50 \times 10^{-4}} = \frac{15 \times 10^{-4}}{50 \times 10^{-4}} = \underline{300 \text{ mT}}$$

Q2) Determine the reluctance of a material with a length of 28 cm and a cross-sectional area of 800 mm² if the relative permeability is 1000. The permeability of vacuum is 4.π.10⁻⁷ Wb/At.m.

[3 marks]

$$R = \frac{l}{\mu_r \mu_0 A} = \frac{0.28}{1000 \times 4\pi \times 10^{-7} \times 800 \times 10^{-6}} = \underline{278.5 \frac{\text{kAt}}{\text{Wb}}}$$

Q3) What is the magnetizing force in 150 turn coil of wire when there are 3A of current through it and the length of the core is 0.2m?

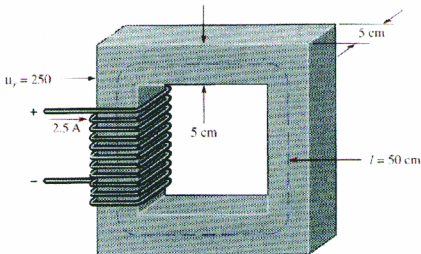
[3 marks]

$$H = \frac{N \times I}{l} = \frac{150 \times 3}{0.2} = \underline{2.25 \text{ kAt/m}}$$

Q4) In the diagram below determine the following:

[3 marks]

- H
- Φ
- B



First, count the number of turns = 10

$$H = \frac{N \times I}{l} = \frac{10 \times 2.5}{0.5} = 50 \text{ At/m}$$

$$R = \frac{l}{\mu_0 \mu_r A} = \frac{0.5}{4\pi \times 10^{-7} \times 250 \times 25 \times 10^{-4}} = 636.6 \frac{\text{kAt}}{\text{Wb}}$$

$$\Phi = \frac{F_m}{R} = \frac{N \times I}{R} = \frac{10 \times 2.5}{636.6 \times 10^3} = 39.3 \mu\text{Wb}$$

$$B = \frac{\Phi}{A} = \frac{39.3 \times 10^{-6}}{25 \times 10^{-4}} = 15.72 \text{ mT}$$

Q5). According to Faraday's Law, what happens to the induced voltage across a given coil if the rate of change of magnetic flux doubles? [1 mark]

The voltage doubles

Q6) A magnetic field is changing at a rate of $3500 \times 10^{-3} \text{ Wb/s}$. How much voltage is induced across a 50 turn coil that is placed in the magnetic field? [1 mark]

$$V = N \left(\frac{d\Phi}{dt} \right) = 50 \times 3500 \times 10^{-3} = 175 \text{ V}$$

Q7) A sine wave has a frequency of 50kHz. How many cycles does it complete in 10ms? [2 marks]

$$T = \frac{1}{f} = \frac{1}{50 \times 10^3} = 20 \mu s$$

$$\text{number of cycles} = \frac{10ms}{T} = \frac{10 \times 10^{-3}}{20 \times 10^{-6}} = 500 \text{ cycles}$$

Q8) Calculate the frequency for a sine wave with a period of 500 μs .

[1 mark]

$$f = \frac{1}{T} = \frac{1}{500 \times 10^{-6}} = 2 \text{ kHz}$$

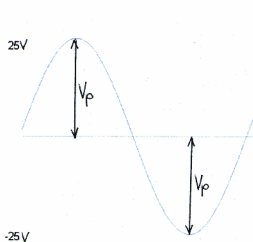
Q9) Calculate the time period for a sine wave with a frequency of 500 MHz.

[1 mark]

$$T = \frac{1}{f} = \frac{1}{500 \times 10^6} = 2 \text{ ns}$$

Q 10) For the sine wave below, determine the peak, peak-to-peak, and RMS values.

[3 marks]



$$V_p = 25V$$

$$V_{pp} = 2 \times 25 = 50V$$

$$\begin{aligned} V_{rms} &= 0.707 \times V_p = \\ &= 0.707 \times 25 = \\ &= 17.675V \end{aligned}$$

Q 11) Convert $\pi/8$ rad to degrees.

[1 mark]

$$\text{degrees} = \left(\frac{180^\circ}{\pi \text{ rad}} \right) \times \text{rad} = \frac{180^\circ}{\pi} \times \frac{\pi}{8} = 22.5^\circ$$

Q 12) Convert 108° into radians.

[1 mark]

$$\text{radians} = \left(\frac{\pi \text{ rad}}{180^\circ} \right) \times \text{degrees} = \frac{\pi}{180^\circ} \times 108^\circ = \frac{3\pi}{5}$$

The result for radians should be expressed as a fraction!

Q13) One sine wave has a positive peak at 75° and another has a positive peak at 100° . How much has each wave shift in phase from the 0° reference? What is the phase angle between them?

[1 mark]

The positive peaks should be occurring at 90° . Therefore, one wave is leading by 15° and the other is lagging by 10° . The phase angle between them is 25°

Q 14) A certain sine wave has a positive going zero crossing at 0° and an rms value of 20V. Calculate its instantaneous value at each of the following angles:

[3 marks]

- 15°
- 110°
- 325°

$$V_p = \frac{V_{rms}}{0.707} = \frac{20}{0.707} = 28.29 \text{ V}$$

$$V(15^\circ) = V_p \sin 15^\circ = 28.29 \times 0.2588 = 7.32 \text{ V}$$

$$V(110^\circ) = V_p \sin 110^\circ = 28.29 \times 0.9397 = 26.58 \text{ V}$$

$$V(325^\circ) = V_p \sin 325^\circ = 28.29 \times (-0.5736) = -16.23 \text{ V}$$

Q 15) For a 0° reference sine wave with an rms value of 6.37V, determine its instantaneous value at each of the following points:

[3 marks]

- $\pi/8$ rad
- $3\pi/4$ rad
- $3\pi/2$ rad

Please, ensure that your calculator is in radian mode!

$$V_p = \frac{V_{rms}}{0.707} = \frac{6.37}{0.707} = 9 \text{ V}$$

$$V(\pi/8) = V_p \sin(\pi/8) = 9 \times 0.383 = 3.44 \text{ V}$$

$$V(3\pi/4) = V_p \sin(3\pi/4) = 9 \times 0.707 = 6.36 \text{ V}$$

$$V(3\pi/2) = V_p \sin(3\pi/2) = 9 \times (-1) = -9 \text{ V}$$

P.S. Don't forget to switch your calculator back to degrees!

:-)

Q 16) Sine wave A lags sine wave B by 30° . Both have peak values of 15V. Sine wave A has reference with positive going at 0° . Determine the instantaneous value of sine wave B at 45° , 200° and 300° . **[3 marks]**

$$B = V_p \sin(45^\circ + 30^\circ) = 15 \sin 75^\circ = 14.49 \text{ V}$$

$$B = V_p \sin(200^\circ + 30^\circ) = 15 \sin 230^\circ = -11.49 \text{ V}$$

$$B = V_p \sin(300^\circ + 30^\circ) = 15 \sin 330^\circ = -7.5 \text{ V}$$

Q 17) How much dc voltage must be added to a 3V rms sine wave in order to make the resulting voltage nonalternating (no negative values)? **[1 mark]**

$$3V_{\text{RMS}} = 3/0.707 = 4.24V_p$$

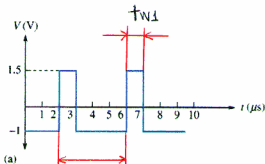
If this DC value is added, the sine wave will never swing below zero.

Q 18) The repetition frequency of a pulse waveform is 2kHz, and the pulse width is $1\mu\text{s}$. What is the percent duty cycle? **[2 marks]**

$$T = \frac{1}{f} = \frac{1}{2 \times 10^3} = 500\mu\text{s}$$

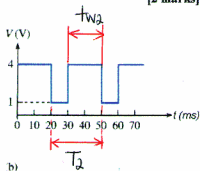
$$\text{duty cycle} = \left(\frac{t_w}{T} \right) \times 100\% = \frac{1 \times 10^{-6}}{500 \times 10^{-6}} \times 100 = 0.2\%$$

Q 19) Determine the duty cycle for each waveform below. **[2 marks]**



$$\begin{aligned} DC_1 &= \left(\frac{t_{w1}}{T_1} \right) \times 100\% = \\ &= \frac{1 \times 10^{-6}}{4 \times 10^{-6}} \times 100\% = \\ &= 25\% \end{aligned}$$

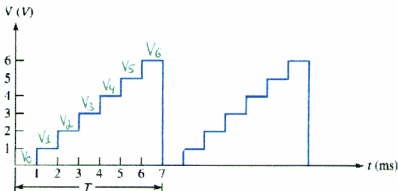
Lubomir Tchernvenkov



$$\begin{aligned} DC_2 &= \left(\frac{t_{w2}}{T_2} \right) \times 100\% = \\ &= \frac{20 \times 10^{-6}}{30 \times 10^{-6}} \times 100\% = \\ &= 66.67\% \end{aligned}$$

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Q 20) A non-sinusoidal waveform called a stair-step is shown below. Determine its average value. [1 mark]



$$V_{AVG} = \frac{V_0 + V_1 + V_2 + V_3 + V_4 + V_5 + V_6}{7} = \frac{0 + 1 + 2 + 3 + 4 + 5 + 6}{7} = \frac{21}{7} = 3V$$

Q 21) What is the fundamental frequency of a square wave with a period of $40\mu s$? [1 mark]

$$f = \frac{1}{T} = \frac{1}{40 \times 10^{-6}} = 25 \text{ kHz}$$

Q 22) A square wave has a period of $40\mu s$. List the first six odd harmonics [2 marks]

$$f = \frac{1}{T} = \frac{1}{40 \times 10^{-6}} = 25 \text{ kHz}$$

$$V_3 = 3 \times 25 \times 10^3 = 75 \text{ kHz}$$

$$V_5 = 5 \times 25 \times 10^3 = 125 \text{ kHz}$$

$$V_7 = 7 \times 25 \times 10^3 = 175 \text{ kHz}$$

$$V_9 = 9 \times 25 \times 10^3 = 225 \text{ kHz}$$

$$V_{11} = 11 \times 25 \times 10^3 = 275 \text{ kHz}$$

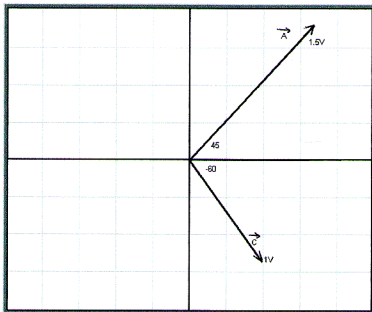
$$V_{13} = 13 \times 25 \times 10^3 = 325 \text{ kHz}$$

Q 23) What value capacitor is capable of storing 10mJ of energy with 100V across its plates? [1 mark]

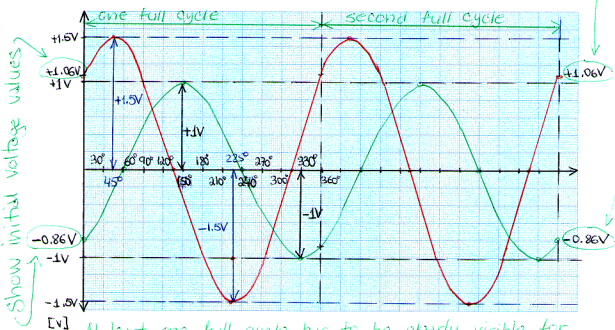
$$W = \frac{1}{2} CV^2 = \frac{CV^2}{2}$$

$$C = \frac{2W}{V^2} = \frac{2 \times 10 \times 10^{-3}}{100^2} = 2 \mu F$$

Q 24) Draw the sine waves represented by the phasors A and C on the same coordinate system, given below. The phasor lengths represent peak values. **[3 marks]**



Initial voltage values and end voltage values should be the same



Q 25) Determine the frequency for the angular velocity of 1256 rad/s

[1 mark]

$$f = \frac{\omega}{2\pi} = \frac{1256}{2\pi} \approx \underline{200 \text{ Hz}}$$

Q 26) Determine the angular velocity for frequency of 2 kHz.

[1 mark]

$$\omega = 2\pi f = 2\pi \times 2 \times 10^3 = \underline{12566 \text{ rad/s}}$$

Q 27) The frequency of a sine wave with 0° phase shift is 5kHz. The peak value of the sine wave is 1 V. Determine the instantaneous value of the sine wave at $30\mu\text{s}$, $75\mu\text{s}$ and $125\mu\text{s}$, measured from the initial positive going zero crossing.

[3 marks]

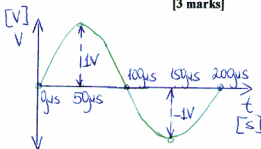
$$T = \frac{1}{f} = \frac{1}{5 \times 10^3} = 200\mu\text{s}$$

We have to convert μs to deg.

$$\text{degrees } (30\mu\text{s}) = \frac{30\mu\text{s}}{200\mu\text{s}} \times 360^\circ = 54^\circ$$

$$\text{degrees } (75\mu\text{s}) = \frac{75}{200} \times 360^\circ = 135^\circ$$

$$\text{degrees } (125\mu\text{s}) = \frac{125}{200} \times 360^\circ = 225^\circ$$



$$V(30\mu\text{s}) = V_p \cdot \sin 54^\circ = 1 \times 0.809 = \underline{0.81V}$$

$$V(75\mu\text{s}) = V_p \cdot \sin 135^\circ = 1 \times 0.707 = \underline{0.71V}$$

$$V(125\mu\text{s}) = V_p \cdot \sin 225^\circ = 1 \times (-0.707) = \underline{-0.71V}$$

Q 28) A mica capacitor has a plate area of 40 cm^2 and a dielectric thickness of 8 mm. What is its capacitance? The dielectric constant of mica is 5.0. The absolute permittivity of vacuum is $8.85 \times 10^{-12} \text{ F/m}$.

[3 marks]

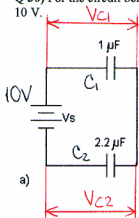
$$C = \epsilon_0 \epsilon_r \frac{A}{d} = 8.85 \times 10^{-12} \times 5 \frac{40 \times 10^{-4}}{8 \times 10^{-3}} = \underline{22.12 \text{ pF}}$$

Q 29) Five 1000 pF capacitors are connected in series. What is the total capacitance?

[1 mark]

$$C_T = \frac{C_x}{5} = \frac{1000}{5} = \underline{200 \text{ pF}}$$

Q 30) For the circuit below, determine the voltage across each capacitor if the value of the battery is 10 V. [2 marks]



Label the circuit before you commence your calculations.

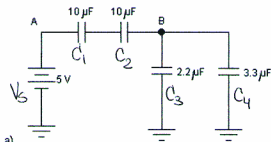
$$C_T = \frac{C_1 C_2}{C_1 + C_2} = \frac{1 \times 2.2}{1 + 2.2} = 0.6875 \mu F$$

$$V_{C1} = \left(\frac{C_T}{C_1} \right) V_S = \left(\frac{0.6875}{1} \right) \times 10 = 6.88 V$$

$$V_{C2} = \left(\frac{C_T}{C_2} \right) V_S = \left(\frac{0.6875}{2.2} \right) \times 10 = 3.12 V$$

$V_S = 10.00 V$
Double checking

Q 31) What is the voltage between points A and B in the circuit below? [3 marks]



Label all components first. You may need to simplify and redraw the circuit.

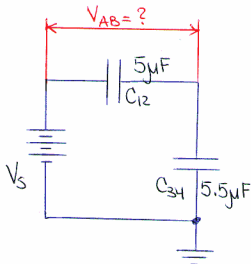
$$C_{12} = \frac{C_1}{2} = \frac{C_2}{2} = \frac{10}{2} = 5 \mu F$$

$$C_{34} = C_3 + C_4 = 2.2 + 3.3 = 5.5 \mu F$$

$$C_T = \frac{C_{12} \cdot C_{34}}{C_{12} + C_{34}} = \frac{5 \times 5.5}{5 + 5.5} = 2.62 \mu F$$

$$V_{AB} = \left(\frac{C_T}{C_{12}} \right) V_S = \left(\frac{2.62}{5} \right) \times 5$$

$$V_{AB} = 2.62 V$$



Q 32) Determine the time constant for RC circuit, where $R = 100\Omega$, $C = 1\mu\text{F}$

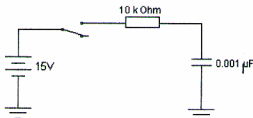
[1 mark]

$$\tau = RC = 100 \times 1 \times 10^{-6} = 100 \times 10^{-6} \text{ s} = \underline{100 \mu\text{s}}$$

Q 33) In the circuit below, the capacitor is initially uncharged. Determine the capacitor voltage at the following times after the switch is closed:

[3 marks]

- $10\mu\text{s}$
- $30\mu\text{s}$
- $50\mu\text{s}$



$$\tau = RC = 10 \times 10^3 \times 1 \times 10^{-9} = 10 \times 10^{-6}$$

$$\tau = 10 \mu\text{s}$$

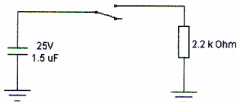
$$\begin{aligned} V(10\mu\text{s}) &= V_F \left(1 - e^{-\frac{10 \times 10^{-6}}{\tau}}\right) = \\ &= 15 \left(1 - e^{-\frac{10 \times 10^{-6}}{10 \times 10^{-6}}}\right) = \\ &= \underline{9.48\text{V}} \end{aligned}$$

$$V(30\mu\text{s}) = 15 \left(1 - e^{-\frac{30 \times 10^{-6}}{10 \times 10^{-6}}}\right) = \underline{14.25\text{V}}$$

$$V(50\mu\text{s}) = 15 \left(1 - e^{-\frac{50 \times 10^{-6}}{10 \times 10^{-6}}}\right) = \underline{14.90\text{V}}$$

Q 34) On the diagram below the capacitor is initially fully charged to 25 V. Determine how long it will take for it to discharge to 3V, after the switch is closed.

[1 mark]



The formula for discharging capacitor is:

$$v = V_i e^{-\frac{t}{RC}}$$

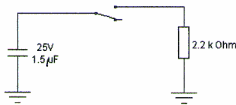
Transposing for t yields a natural logarithm \ln .

$$t = -RC \ln\left(\frac{v}{V_i}\right) =$$

$$= -2.2 \times 10^3 \times 1.5 \times 10^{-6} \ln\left(\frac{3}{25}\right) \approx \underline{7\text{ms}}$$

Q 35) In the figure below, the capacitor is initially fully charged. Determine the capacitor voltage at the following times after the switch is closed: **[3 marks]**

- 5ms
- 10ms
- 15ms



$$\tau = RC = 2.2 \times 10^3 \times 1.5 \times 10^{-6}$$

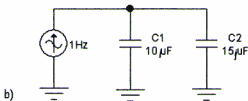
$$\tau = 3.3 \text{ ms}$$

$$V(5\text{ms}) = V_i e^{-\frac{t_1}{\tau}} = 25 \cdot e^{-\frac{5}{3.3}} = 5.5 \text{ V}$$

$$V(10\text{ms}) = V_i e^{-\frac{t_2}{\tau}} = 25 \cdot e^{-\frac{10}{3.3}} = 1.2 \text{ V}$$

$$V(15\text{ms}) = V_i e^{-\frac{t_3}{\tau}} = 25 \cdot e^{-\frac{15}{3.3}} = 0.3 \text{ V}$$

Q 36) What is the value of the total capacitive reactance of the circuit below? **[1 mark]**



$$C_{12} = C_1 + C_2 = 10 + 15$$

$$C_{12} = 25 \mu\text{F}$$

$$X_{C1} = \frac{1}{2\pi f C_{12}} = \frac{1}{2\pi \times 1 \times 25 \times 10^{-6}} = 6.366 \text{ k}\Omega$$

Q 37) In the previous question Q36, what frequency is required to produce an X_C of 100Ω ? **[1 mark]**

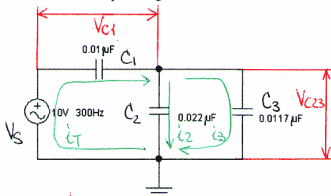
$$f = \frac{1}{2\pi C_{12} X_{C12}} = \frac{1}{2\pi \times 25 \times 10^{-6} \times 100} = 63.63 \text{ Hz}$$

Q 38) A 1 kHz voltage is applied to a $47\mu\text{F}$ capacitor, and 1mA of rms current is measured. What is the value of the voltage? [2 marks]

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 1000 \times 47 \times 10^{-6}} = 3.38 \Omega$$

$$V = i X_C = 1 \times 10^{-3} \times 3.38 = 3.38 \text{ mV}$$

Q 39) Determine the ac voltage across each capacitor and the current in each branch of the circuit below. What is the phase angle between the current and the voltage in each case? [5 marks]



Label the circuit first.

$$C_{23} = C_2 + C_3 = 22 + 11.7 = 33.7 \text{ nF}$$

$$C_T = \frac{C_1 \times C_{23}}{C_1 + C_{23}} = \frac{10 \times 33.7}{10 + 33.7} = 7.71 \text{ nF}$$

$$X_{C1} = \frac{1}{2\pi f C_1} = \frac{1}{2\pi \times 300 \times 10 \times 10^{-9}} = 53.05 \text{ k}\Omega$$

$$X_{C2} = \frac{1}{2\pi f C_2} = \frac{1}{2\pi \times 300 \times 22 \times 10^{-9}} = 24.11 \text{ k}\Omega$$

$$X_{C3} = \frac{1}{2\pi f C_3} = \frac{1}{2\pi \times 300 \times 11.7 \times 10^{-9}} = 45.34 \text{ k}\Omega$$

$$X_{CT} = \frac{1}{2\pi f C_T} = \frac{1}{2\pi \times 300 \times 7.71 \times 10^{-9}} = 68.81 \text{ k}\Omega$$

$$i_T = \frac{V_S}{X_{CT}} = \frac{10}{68810} = 145.3 \mu\text{A}$$

$$V_{C1} = i_T \times X_{C1} = 145.3 \times 10^{-3} \times 53050 = 7.7 \text{ V}$$

$$V_{C23} = V_S - V_{C1} = 10 - 7.7 = 2.3 \text{ V}$$

$$i_2 = \frac{V_{C23}}{X_{C2}} = \frac{2.3}{24110} = 95.4 \mu\text{A}$$

$$i_3 = \frac{V_{C23}}{X_{C3}} = \frac{2.3}{45340} = 50.7 \mu\text{A}$$

The current through the capacitors always leads the voltage by 90° .

Q 40) Fifty volts are induced across a 25mH coil. At what rate is the current changing. [1 mark]

$$V = L \frac{di}{dt} \therefore$$

$$\frac{di}{dt} = \frac{V}{L} = \frac{50}{0.025} = 2 \text{ kHz}$$

Q 41) How many turn are required to produce 30mH with a coil wound up on a cylindrical core having a cross-sectional area of 100 mm² and a length of 50 mm? The core has a permeability of $1.2 \times 10^{-6} \text{ H/m}$. [2 marks]

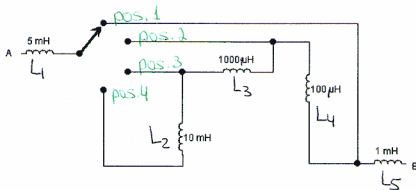
$$L = \frac{N^2 \mu A}{l}$$

$$N = \sqrt{\frac{L \times l}{\mu A}} = \sqrt{\frac{0.03 \times 0.05}{1.2 \times 10^{-6} \times 100 \times 10^{-6}}} = 3535.5 \text{ turns}$$

Q 42) How much energy is stored by a 100mH inductor with a current of 1A. [1 mark]

$$W = \frac{1}{2} LI^2 = \frac{1}{2} \times 0.1 \times (1)^2 = 50 \text{ mJ}$$

Q 43) What is the total inductance between points A and B for each switch position below: [4 marks]



Label all components and switch positions first.

$$\text{pos. 1} \rightarrow L_T = L_1 + L_5 = 5 + 1 = 6 \text{ mH}$$

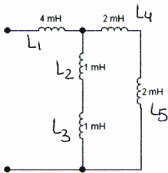
$$\text{pos. 2} \rightarrow L_T = L_1 + L_4 + L_5 = 5 + 0.1 + 1 = 6.1 \text{ mH}$$

$$\text{pos. 3} \rightarrow L_T = L_1 + L_3 + L_4 + L_5 = 5 + 1 + 0.1 + 1 = 7.1 \text{ mH}$$

$$\text{pos. 4} \rightarrow L_T = L_2 + L_3 + L_4 + L_5 = 5 + 10 + 1 + 0.1 + 1 = 17.1 \text{ mH}$$

Q 44) Determine the total inductance of the circuit below:

[1 mark]



$$L_{45} = L_4 + L_5 = 2 + 2 = 4 \text{ mH}$$

$$L_{23} = L_2 + L_3 = 1 + 1 = 2 \text{ mH}$$

$$L_{2345} = \frac{L_{23} \cdot L_{45}}{L_{23} + L_{45}} = \frac{2 \times 4}{2 + 4} = 1.33 \text{ mH}$$

$$L_T = L_{2345} + L_1 = 1.33 + 4 = \underline{5.33 \text{ mH}}$$

Q 45) In a series RL circuit, determine how long it takes for the current to build up to its full value if $R = 56 \Omega$ and $L = 50 \mu\text{H}$.

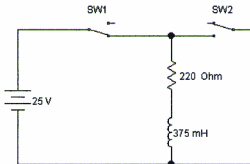
[2 marks]

$$\tau = \frac{L}{R} = \frac{50 \times 10^{-6}}{56} = 893 \text{ ns}$$

$$5\tau = 5 \times 893 \times 10^{-9} = \underline{4.46 \mu\text{s}}$$

Q 46) Initially SW1 is opened. Determine the current through the inductor at 1 ms, 3 ms and 5 ms after the switch SW1 is closed.

[3 marks]



$$\tau = \frac{L}{R} = \frac{0.375}{220} = 1.7 \text{ ms}$$

$$I_F = \frac{V_S}{R} = \frac{25}{220} = 113.63 \text{ mA}$$

$$i(1\text{ms}) = I_F (1 - e^{-\frac{t_1}{\tau}}) = 113.63 \times 10^{-3} (1 - e^{-\frac{1}{1.7}}) = \underline{50.53 \text{ mA}}$$

$$i(3\text{ms}) = I_F (1 - e^{-\frac{t_2}{\tau}}) = 113.63 \times 10^{-3} (1 - e^{-\frac{3}{1.7}}) = \underline{94.17 \text{ mA}}$$

$$i(5\text{ms}) = I_F (1 - e^{-\frac{t_3}{\tau}}) = 113.63 \times 10^{-3} (1 - e^{-\frac{5}{1.7}}) = \underline{107.63 \text{ mA}}$$

Q 47) For the circuit in the previous question, initially the final steady value of the current through the inductor is reached. Then SW1 opens and SW2 closes simultaneously. Determine the current through the inductor at 1ms, 3ms and 5 ms after the switch SW2 is closed. **[3 marks]**

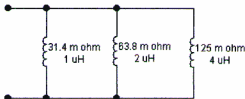
We already know that $\tau = 1.7 \text{ ms}$ and $I_F = 113.63 \text{ mA}$, which now becomes $I_i = 113.63 \text{ mA}$.

$$i(1\text{ms}) = I_i e^{-\frac{t_1}{\tau}} = 113.63 \times 10^{-3} \cdot e^{-\frac{1}{1.7}} = 63.1 \text{ mA}$$

$$i(3\text{ms}) = I_i e^{-\frac{t_2}{\tau}} = 113.63 \times 10^{-3} \cdot e^{-\frac{3}{1.7}} = 19.46 \text{ mA}$$

$$i(5\text{ms}) = I_i e^{-\frac{t_3}{\tau}} = 113.63 \times 10^{-3} \cdot e^{-\frac{5}{1.7}} = 6 \text{ mA}$$

Q 48) What is the value of the total inductive reactance of the circuit below, when the frequency is 1MHz? **[2 marks]**



Solution 1:

$$X_{L1} = 2\pi f L_1 = 6.28 \Omega$$

$$X_{L2} = 2\pi f L_2 = 12.57 \Omega$$

$$X_{L3} = 2\pi f L_3 = 25.13 \Omega$$

$$\frac{1}{X_{LT}} = \frac{1}{X_{L1}} + \frac{1}{X_{L2}} + \frac{1}{X_{L3}} = \frac{1}{6.28} + \frac{1}{12.57} + \frac{1}{25.13}$$

$$X_{LT} = 3.588 \Omega$$

Q 49) Five inductors are connected in series. The lowest value is 5 μH . If the value of each inductor is twice that of the preceding one, and if the inductors are connected in order of ascending values, what is the total inductance? **[1 mark]**

$$L_1 = 5 \mu\text{H} ; L_2 = 10 \mu\text{H} ; L_3 = 20 \mu\text{H} ; L_4 = 40 \mu\text{H} ; L_5 = 80 \mu\text{H}$$

$$L_T = L_1 + L_2 + L_3 + L_4 + L_5 =$$

$$= 5 + 10 + 20 + 40 + 80$$

$$L_T = 155 \mu\text{H}$$

Solution 2:

$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

$$\frac{1}{L_T} = \frac{1}{1} + \frac{1}{2} + \frac{1}{4}$$

$$L_T = 571 \text{ nH}$$

$$\begin{aligned} X_{LT} &= 2\pi f L_T = \\ &= 2\pi \times 1 \times 10^6 \times 571 \times 10^{-9} = \\ &= 3.588 \Omega \end{aligned}$$

On the actual test you need to present one method only, but if you know two, you can double-check.

Q 50) Determine the ac voltage across each inductor and the current in each branch of the circuit below. What is the phase angle between the current and the voltage in each case? [5 marks]



First, clearly label everything!

$$L_{23} = \frac{L_2 \times L_3}{L_2 + L_3} = \frac{20 \times 40}{20 + 40} = 13.33 \mu\text{H}$$

$$L_T = L_1 + L_{23} = 50 + 13.33 = 63.33 \mu\text{H}$$

$$X_{LT} = 2\pi f L_T = 2\pi \times 2500 \times 63.33 \times 10^{-6} = 0.995 \Omega$$

$$i_T = \frac{V_s}{X_{LT}} = \frac{10}{0.995} = 10.05 \text{ A}$$

$$\begin{aligned} i_2 &= 2i_3 \\ i_2 + i_3 &= 10.05 \end{aligned} \quad \leftarrow \text{System of two equations with two variables. The method is identical to what is already covered in "DC Fundamentals Trial Mid-Semester Test", p.10}$$

$$2i_3 + i_3 = 10.05$$

$$i_3 = \frac{10.05}{3} = 3.35 \text{ A}$$

$$i_2 = 2 \times 3.35 = 6.7 \text{ A}$$

$$V_{L1} = \left(\frac{L_1}{L_T} \right) V_s = \left(\frac{50}{63.33} \right) \times 10 = 7.89 \text{ V}$$

$$V_{L23} = V_s - V_{L1} = 10 - 7.89 = 2.11 \text{ V}$$

All currents lag voltages by 90° .

END OF TEST
(Check your work!)

P.S. If you have time on the test, please, always double-check your work!