ELECTRICAL MATHEMATICS
TEST 4 – *TRIAL TEST/ASSIGNMENT*

Notes:

- Test covers simultaneous equations, factorising, solving and graphing quadratic equations.
- The actual test will be closed book, with calculator and ruler required.
- It is **ESSENTIAL** to show working/steps, where asked, otherwise **no marks** can be given.

1. Factorise the following:

a. $4x^2 - 6x$ $= 2x(2x-3)$ → Common Factor

b. $b^2 - 16$ $= b^2 - 4^2 = (b-4)(b+4)$ → Difference of Squares

c. $F^2 + 13F + 12$ $= (F+1)(F+12)$ → Trinomial (simple)

d. $4a^2 + 28a - 15$

$$\frac{(4a-2)(4a+30)}{4} = \frac{(4a-2)(4a+30)}{2\times 2} =$$

Trinomial (harder)

$$= \frac{4a-2}{2} \times \frac{4a+30}{2} =$$

$$= (2a-1)(2a+15)$$

2. Given that the roots/solutions of the quadratic equation $ax^2 + bx + c = 0$ are

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

$a=2, b=-6, c=+3$

solve $2y^2 - 6y + 3 = 0$, giving the roots correct to three significant figures.

$$y_{12} = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \times 2 \times 3}}{2 \times 2} =$$

$$= \frac{6 \pm \sqrt{36-24}}{4} = \frac{6 \pm \sqrt{12}}{4}$$

$$y_1 = \frac{6 + \sqrt{12}}{4} = \frac{6 + 3.46}{4} = 2.37 \rightarrow$$

$$y_2 = \frac{6 - \sqrt{12}}{4} = \frac{6 - 3.46}{4} = 0.635 \rightarrow$$

Double check: $2(2.37)^2 - 6(2.37) + 3 =$

$= 11.23 - 14.22 + 3 \approx 0$

$2(0.635)^2 - 6(0.635) + 3 =$

$= 0.806 - 3.81 + 3 \approx 0$

3. For each of the following, determine the axis of symmetry, turning point (vertex) coordinates and the y-intercept (ie **three** quantities for each equation):

   a. $y = x^2 + 8x + 4$

   b. $y = -2x^2 + 4x + 6$

a.) $y = x^2 + 8x + 4$ $\quad (ax^2 + bx + c)$

$\therefore a = 1, b = 8, c = 4$

Axis of symmetry $= \frac{-b}{2a} = \frac{-8}{2 \times 1} = -4 \rightarrow$

Turning point (vertex) $= (-4)^2 + 8(-4) + 4 = -12 \rightarrow$

y-intercept $= c = 4 \rightarrow$

b.) $y = -2x^2 + 4x + 6$

$\therefore a = -2, b = 4, c = 6$

Axis of symmetry $= \frac{-b}{2a} = \frac{-4}{2(-2)} = \frac{-4}{-4} = 1 \rightarrow$

Vertex $= -2(1)^2 + 4(1) + 6 = 8 \rightarrow$

y-intercept $= 6 \rightarrow$

Or we can simplify: $y = -x^2 + 2x + 3$, or

$y = x^2 - 2x - 3$

$y = (x+1)(x-3)$

$x_1 = -1$, $x_2 = 3$ } x-intercepts

4. **Sketch** the curve for each of the equations in Q3, clearly labelling the key features. (Use the graph paper provided on p.6).

See p.6

5. Solve each of the following quadratic equations by factoring:

a. $5f - f^2 = 0$

$f(5-f) = 0$

$\therefore f_1 = 0 \rightarrow$ or $(5-f) = 0$

$\therefore f_2 = 5 \rightarrow$

b. $x^2 - 7x - 18 = 0$

$(x-9)(x+2) = 0$

$\therefore (x-9) = 0$ or $(x+2) = 0$

$x_1 = 9 \rightarrow$ $x_2 = -2 \rightarrow$

c. $4a^2 - 16a + 15 = 0$

$$\frac{(4a-6)(4a-10)}{4} = 0$$

$$\frac{(4a-6)}{2} \times \frac{(4a-10)}{2} = 0$$

$$(2a-3) \times (2a-5) = 0$$

$\therefore (2a-3) = 0$ or $(2a-5) = 0$

$2a = 3$

$a_1 = \frac{3}{2} = 1\frac{1}{2}$

$2a = 5$

$a_2 = \frac{5}{2} = 2\frac{1}{2}$

P.S. Don't forget to double-check your results!

6. Solve (algebraically) for x and y, in each of the simultaneous equation pairs below:

a. $y = x^2 - 4x - 12$ → equation ①
$y = -3x + 5$ → equation ②

Equation ① must be equal to equation ②, so:

$x^2 - 4x - 12 = -3x + 5$

$x^2 - 4x + 3x - 12 - 5 = 0$

$x^2 - x - 17 = 0$

$$x_{12} = \frac{1 \pm \sqrt{1 - 4(-17)}}{2}$$

$x_1 = 4.66$, $x_2 = -3.65$

$y_1 = -3(4.66) + 5 = -8.98$, $y_2 = -3(-3.65) + 5 = 15.95$

$\begin{cases} x_1 = 4.66 \\ y_1 = -8.98 \end{cases}$ $\begin{cases} x_2 = -3.65 \\ y_2 = 15.95 \end{cases}$

b. $y = x^2 - 2x - 3$ → equation ①
$2y - x + 2 = 0$ → equation ②

Solving by substitution. Substitute equation ① into ②.

$2(x^2-2x-3) - x + 2 = 0$

$2x^2 - 4x - 6 - x + 2 = 0$

$2x^2 - 5x - 4 = 0$

$$x_{12} = \frac{5 \pm \sqrt{25 - 4(-8)}}{4}$$

$x_1 = 3.14$, $x_2 = -0.637$

Now substitute each x value into equation ①.

$y_1 = (3.14)^2 - 2(3.14) - 3 = 0.580$

$y_2 = (-0.637)^2 - 2(-0.637) - 3 = -1.32$

$\begin{cases} x_1 = 3.14 \\ y_1 = 0.580 \end{cases}$ $\begin{cases} x_2 = -0.637 \\ y_2 = -1.32 \end{cases}$

P.S. Always check your solutions before submitting your work!

7. Solve, by determinants, the following simultaneous equations:

$4x - 3y = 18$
$x + 2y = -1$

Equations are in 'standard' format, so:

$$\Delta = \begin{vmatrix} +4 & -3 \\ +1 & +2 \end{vmatrix} = (4 \times 2) - [1 \times (-3)] = 8 - (-3) = 11$$

$$\Delta_x = \begin{vmatrix} 18 & -3 \\ -1 & 2 \end{vmatrix} = (18 \times 2) - [(-1) \times (-3)] = 36 - 3 = 33$$

$$\Delta_y = \begin{vmatrix} 4 & 18 \\ 1 & -1 \end{vmatrix} = [4 \times (-1)] - (1 \times 18) = -4 - 18 = -22$$

$$x = \frac{\Delta x}{\Delta} = \frac{33}{11} = 3$$

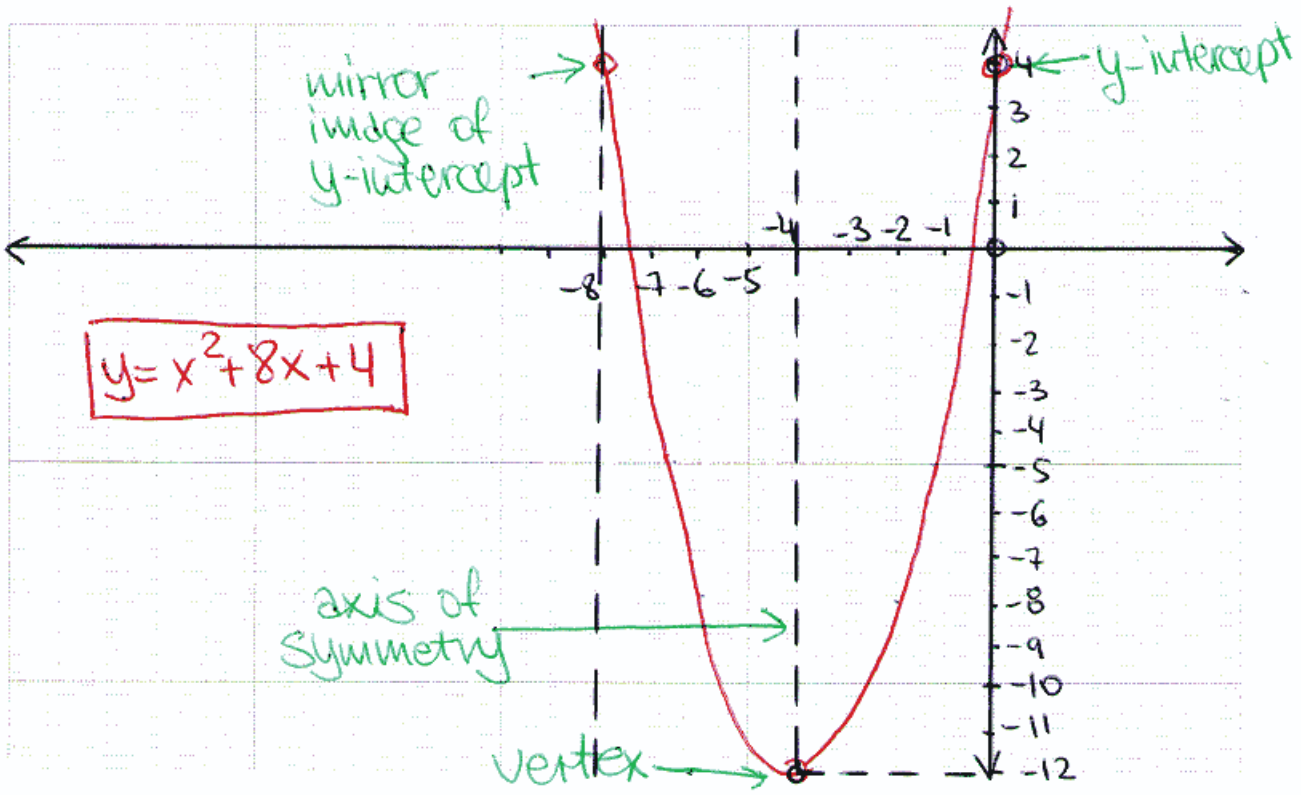
$$y = \frac{\Delta y}{\Delta} = \frac{-22}{11} = -2$$

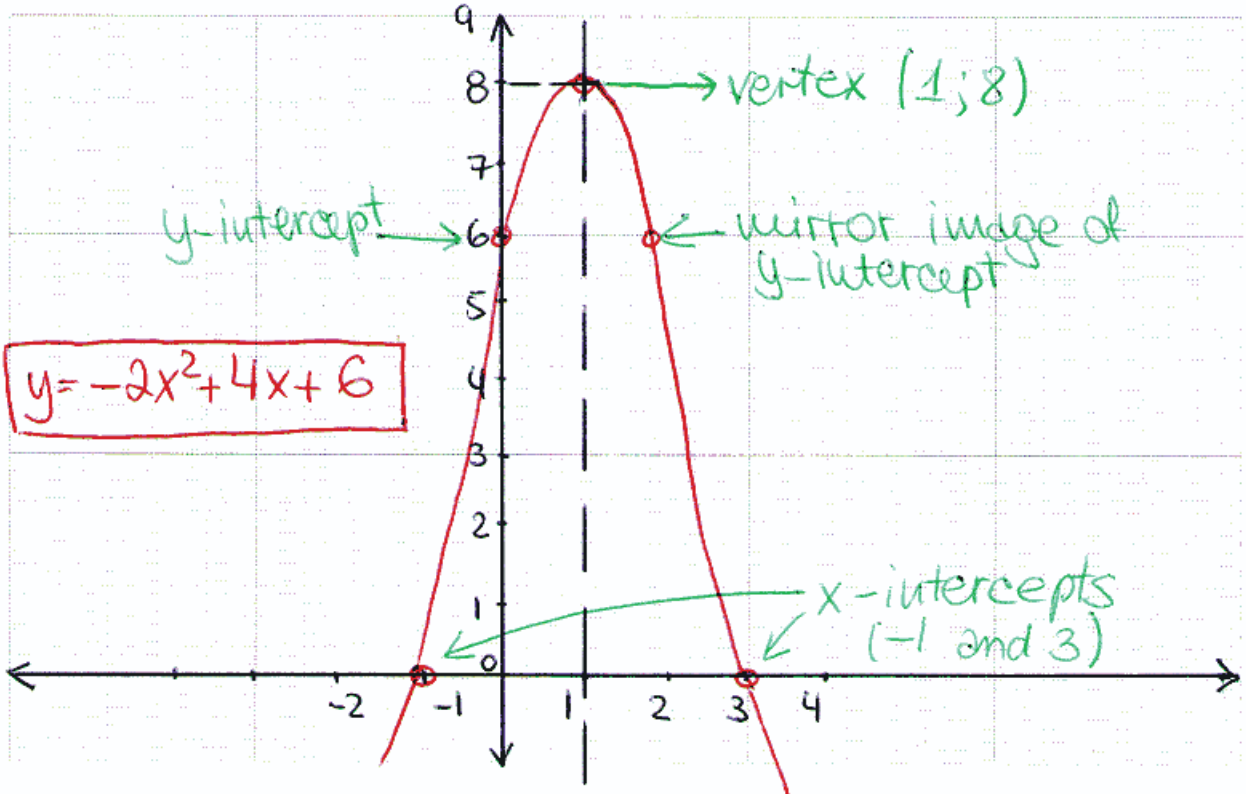
Possible ways to check your solutions:
① Substitute the results back into the original equations
② Use graphic calculator
③ Use the computational knowledge engine: "Wolfram Alpha"

Solution of simultaneous equations using determinants

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2$$

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = \frac{c_1b_2 - c_2b_1}{a_1b_2 - a_2b_1} \quad \text{and} \quad y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = \frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}$$



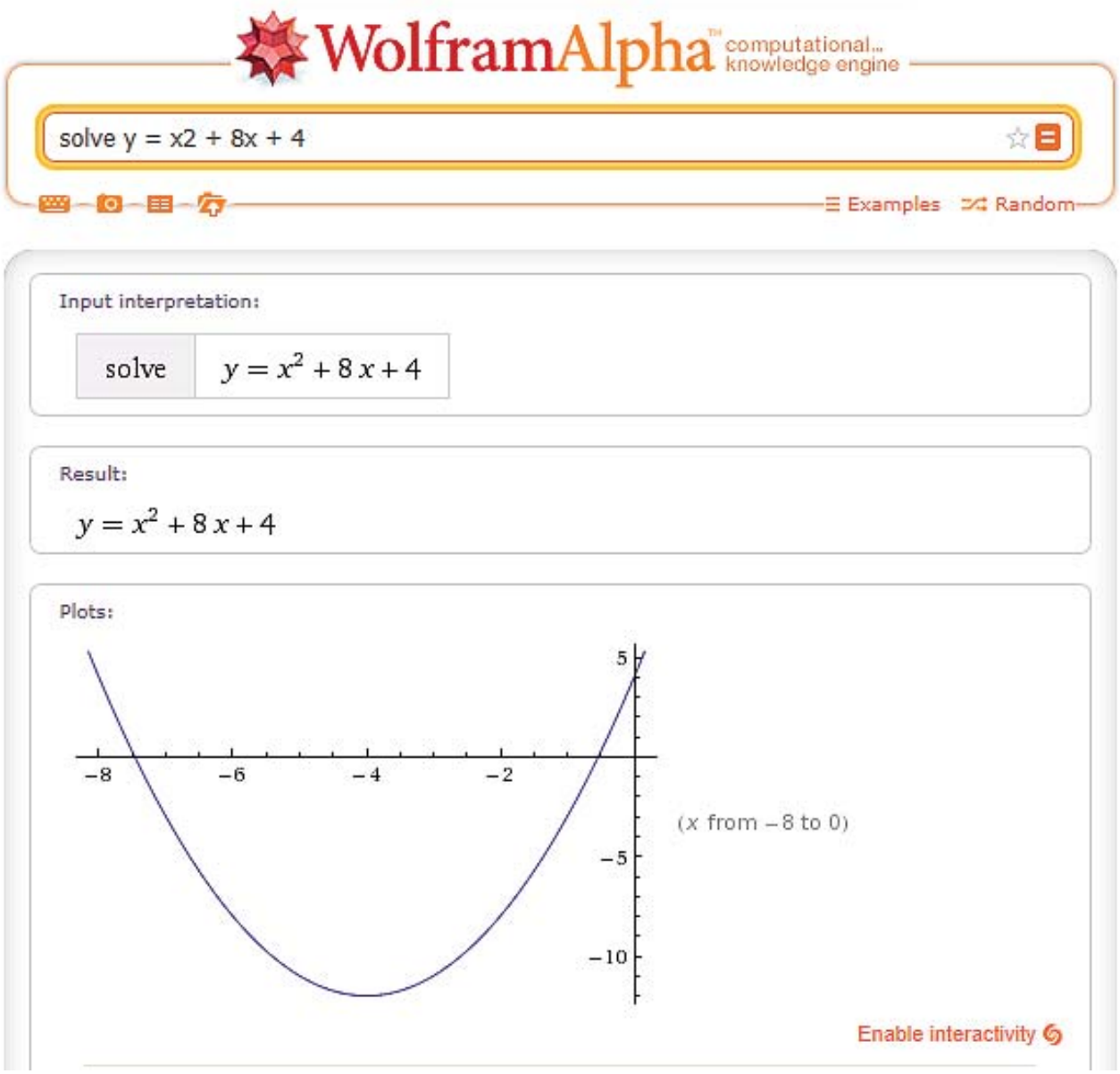


------- **END OF TRIAL TEST/ASSIGNMENT** - ***Check your work!*** -------

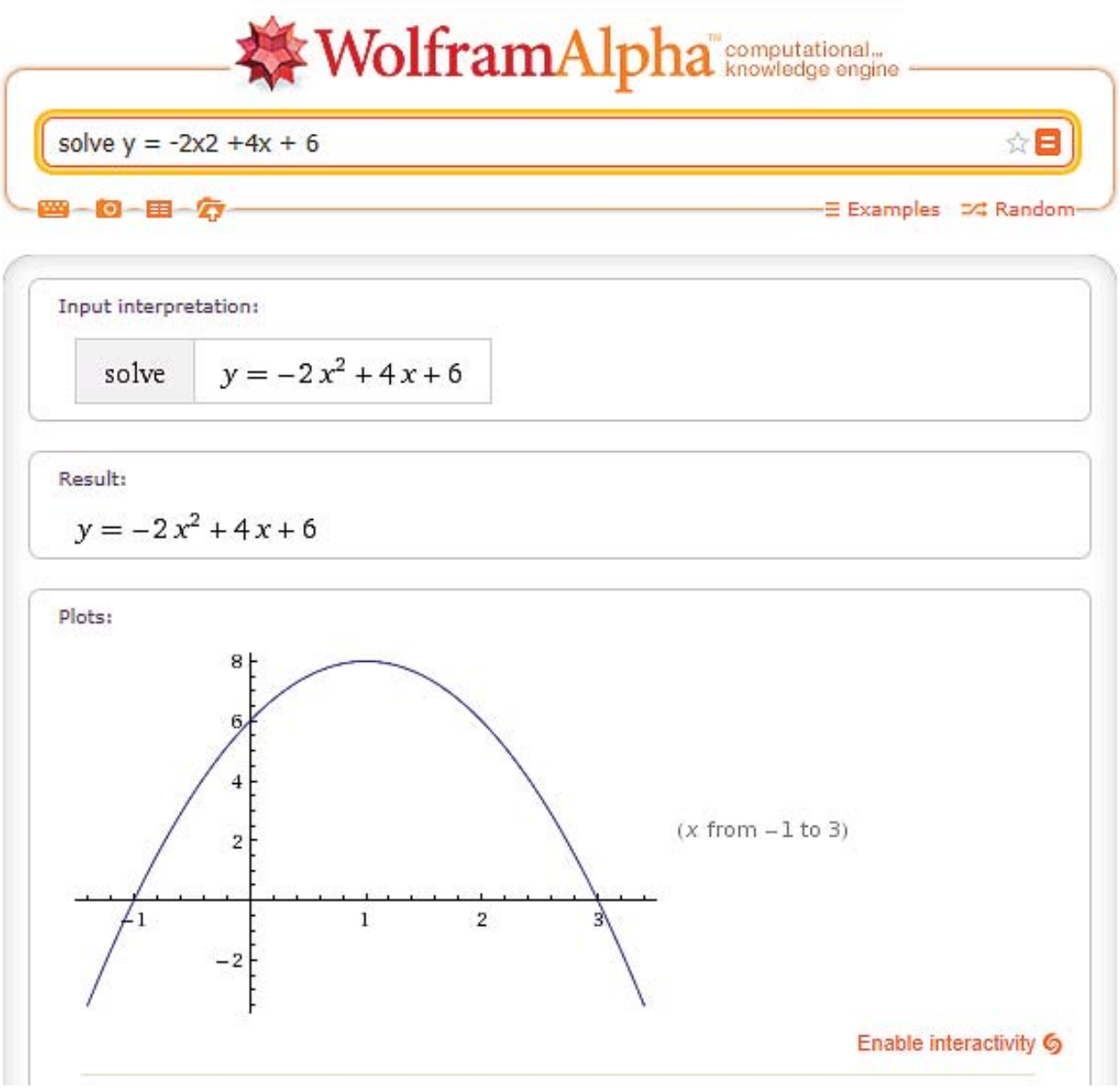
APPENDIX A

One possible way to check your solutions is with the computational knowledge engine "Wolfram Alpha" (http://www.wolframalpha.com/)

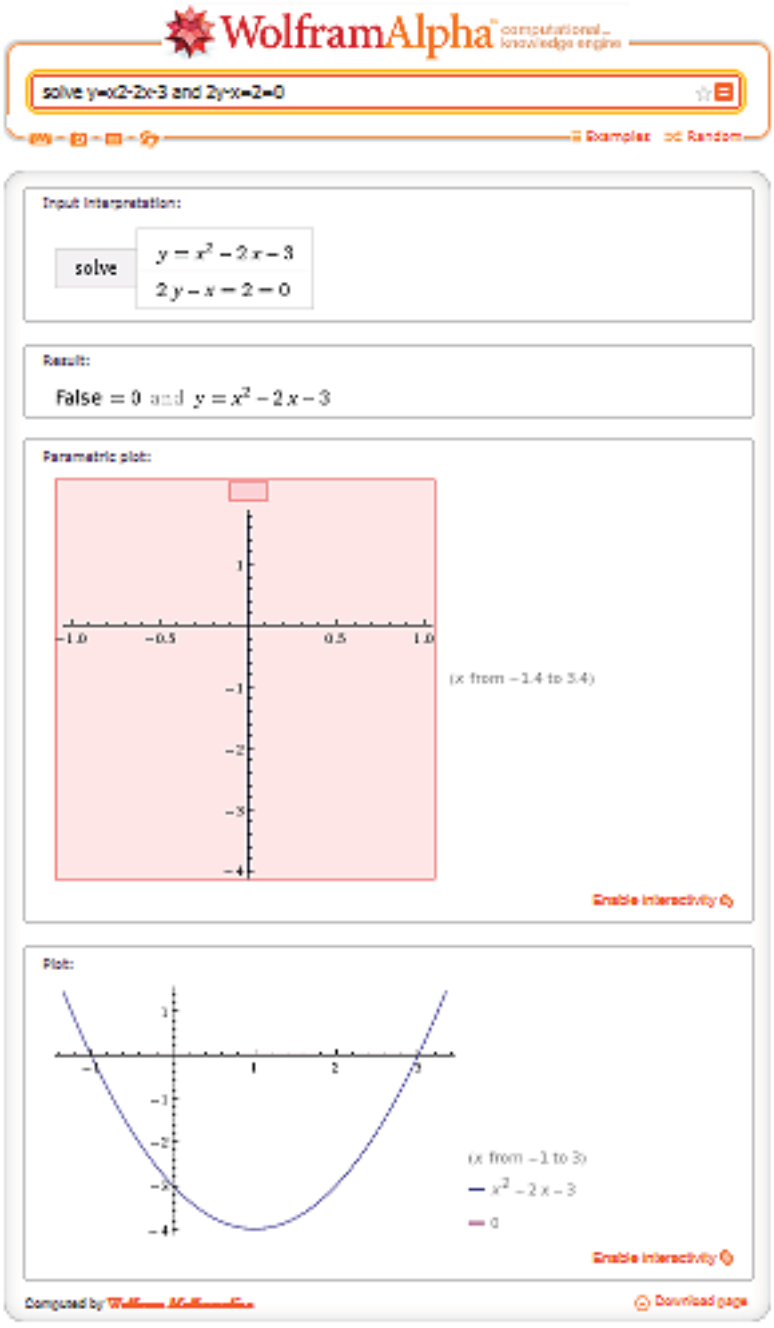
For example for question 3a just type the equation into the search bar and you will obtain this result:



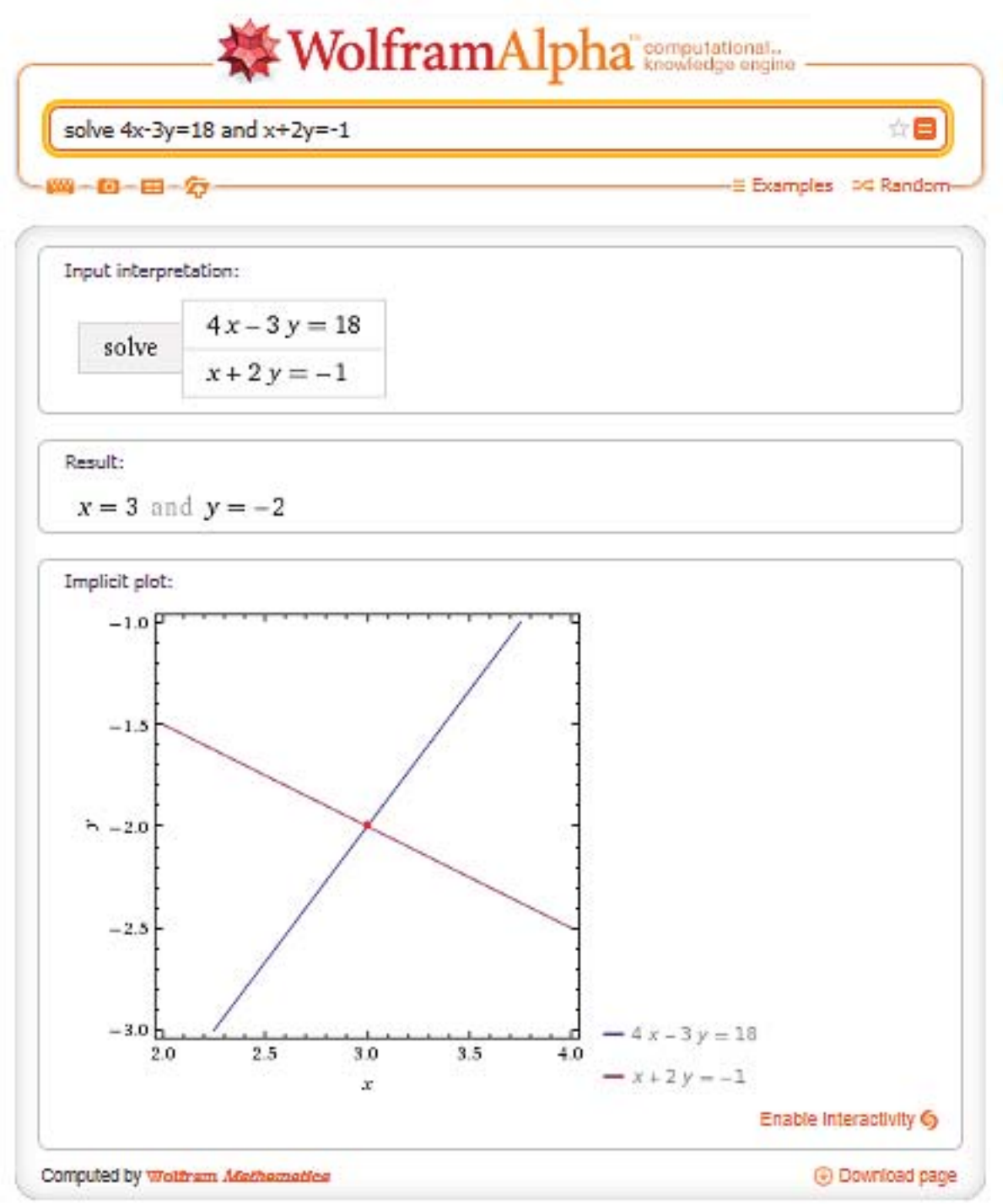
Then for 3b the following result is obtained:



Wolfram Alpha allows us to also solve simultaneous equations. For example for question 6b we get:



And for question 7 the answer is:



Mind you, while this scientific, computational knowledge engine could be very useful during your preparation study for the test, you will not be allowed to use a computer or too sophisticated calculator on the actual test. The point is to learn how to obtain and draw the results yourself!