ELECTRICAL MATHEMATICS
TEST 5 – *TRIAL TEST/ASSIGNMENT*

Notes:

- Test covers trigonometric functions, logarithms and exponentials, complex numbers and vectors.
- The actual test will be closed book, with calculator and ruler required.
- It is **ESSENTIAL** to show working/steps, where asked, otherwise **no marks** can be given.

1. Given that π radians = 180°, express:

$360° = 2\pi \text{ (rad)}$
$\therefore 1° = \frac{\pi}{180} \text{ (rad)}$

1.1 55° in radians

$55° \times \frac{\pi}{180} = 0.96 \text{ (rad)}$

$2\pi \text{ (rad)} = 360°$
$1 \text{ (rad)} = \frac{180°}{\pi}$

1.2 $\frac{2}{3}\pi$ radians as degrees

$\frac{2}{3}\pi \text{ (rad)} = \frac{2\pi}{3} \times \frac{180}{\pi} = 120°$

1.3 45° in radians (expressed **in terms of π**)

$45° = 45 \frac{\pi}{180} = \frac{\pi}{4}$

1.4 1.7 radians as degrees

$1.7 \text{ (rad)} = 1.7 \frac{180}{\pi} = 97.4°$

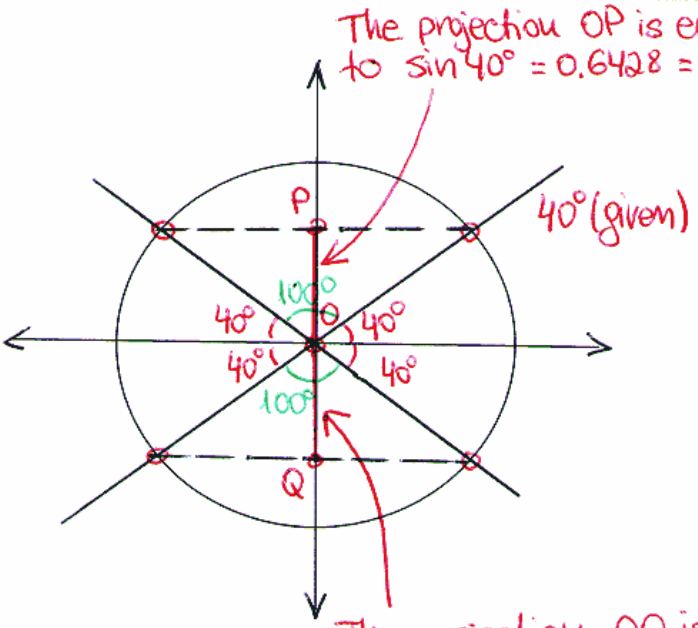
2. Use your calculator to determine the trigonometric ratios of the following, giving answers to 3 significant figures:

2.1 $\tan 21°$	0.384
2.2 $\cos 225°$	−0.707
2.3 $\sin \frac{3\pi}{4}$ rad	0.707
2.4 $\cos\left(\frac{4}{3}\pi \text{ rad}\right)$	−0.5
2.5 $\tan(-21°)$	−0.384

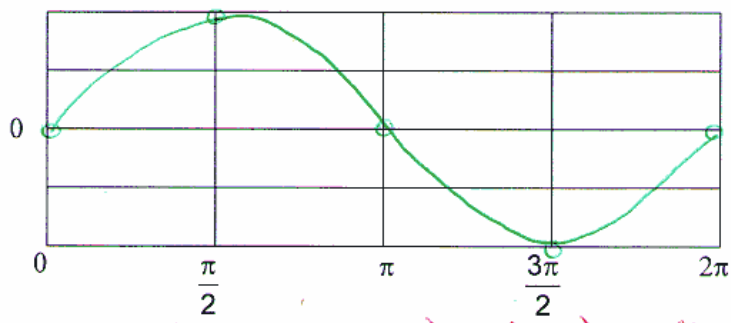
Be careful! Calculator must be set in radian mode.

3. Given that sin 40° = 0.6428, use the unit circle (NOT calculator) to determine:

3.1	sin 140°	0.6428 (same as sin 40°)
3.2	sin 220°	−0.6428 (same as sin (−40°))
3.3	sin 320°	−0.6428 (same as sin (−40°))



4. Sketch the graph of $y = \sin\theta$ for $0 \le \theta \le 2\pi$



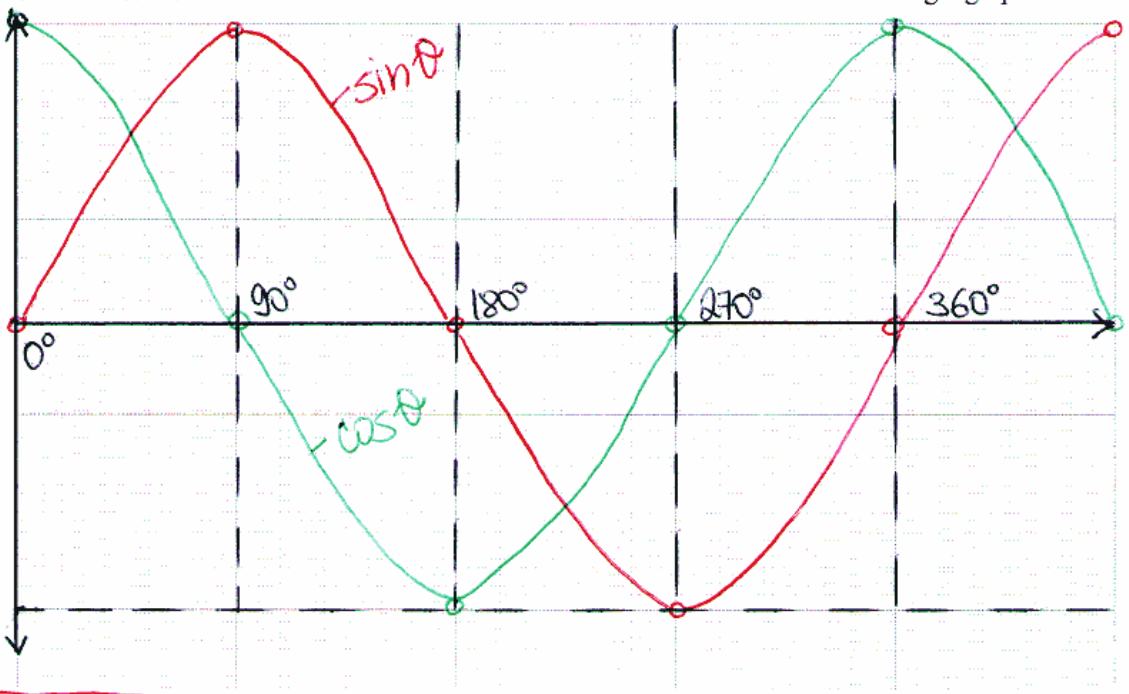
$\sin 0(\text{rad}) = \sin 0° = 0$
$\sin \frac{\pi}{2} = \sin 90° = 1$
$\sin \pi = \sin 180° = 0$
$\sin \frac{3\pi}{2} = \sin 270° = -1$
$\sin 2\pi = \sin 360° = 0$

5. State the amplitude of the following functions:

5.1 $y = 4 \sin 4\theta$	4
5.2 $y = 0.4 \cos 12\theta$	0.4

$y = A \sin\theta$ $\quad$ $y = A \cos\theta$
A: amplitude (peak)

6. Illustrate the main difference between a sine wave and a cosine wave using a graph.



For the sin function, see q.4

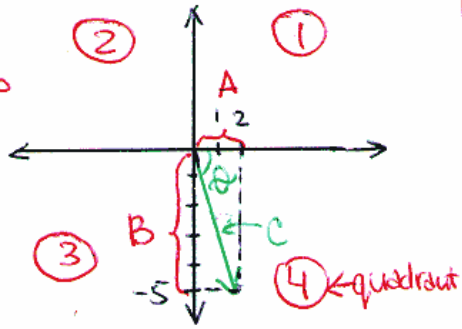
$\cos 0 = 1$, $\cos 90 = 0$, $\cos 180 = -1$, $\cos 270 = 0$, $\cos 360 = 1$,
$\therefore$ cos wave leads the sin wave by 90°

7. Convert the following:

7.1 2 – j5 to polar form $= 5.4 \angle -68°$

$C = \sqrt{2^2 + 5^2} = 5.4$

$\theta = \tan^{-1}\left(\frac{-5}{2}\right) = -68°$



Conversion Formulas Polar to Rectangular:
$C\angle\theta = A + jB$
$A = C \cdot \cos\theta$
$B = C \cdot \sin\theta$

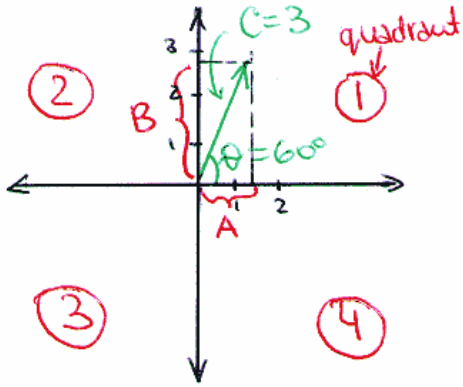
Rectangular to Polar:
$A + jB = C\angle\theta$
$C = \sqrt{A^2 + B^2}$
$\theta = \tan^{-1}\frac{B}{A}$

These formulas only apply for quadrants 1 and 4. For quadrants 2 and 3 (rarely used) you may need to add or subtract 180° for the angle.

7.2 3 ∠60° to rectangular form

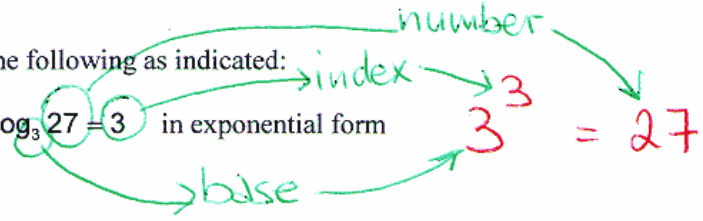
$A = 3 \cdot \cos 60° = 1.5$

$B = 3 \cdot \sin 60° = 2.6$



8. Express the following as indicated:

8.1 $\log_3 27 = 3$ in exponential form



8.2 $\log_2 \frac{1}{4} = -2$ in exponential form

$2^{-2} = \frac{1}{4}$

8.3 $10^m = n$ in logarithmic form

$\log_{10} n = m$

8.4 $8^{-\frac{2}{3}} = \frac{1}{4}$ in logarithmic form

$\log_8 \frac{1}{4} = -\frac{2}{3}$

9. Evaluate:

9.1 $\log_3 81$

Set $x = \log_3 81$

$3^x = 81$

$3^x = 3^4$

$x = 4$

9.2 $\log_e 8.5$

Using a calculator:

$\log_e 8.5 = \ln 8.5 = 2.140$

9.3 $\log_n n$

Set $y = \log_n n$

$n^y = n^1$

$y = 1$

9.4 $\log_5 5^2$

Set $z = \log_5 5^2$

$5^z = 5^2$

$z = 2$

10. Solve the following exponential equations:

10.1 $4^x = 8$

$(2^2)^x = 2^3$

$(2)^{2x} = (2)^3$

$2x = 3$

$x = \frac{3}{2} = 1\frac{1}{2}$

10.2 $\frac{1}{4^a} = 8$

$4^{-a} = 8$

$(2^2)^{-a} = 2^3$

$2^{-2a} = 2^3$

$-2a = 3$

$a = -\frac{3}{2} = -1\frac{1}{2}$

10.3 $9 = 27^{2-x}$

$3^2 = (3^3)^{2-x}$

$3^2 = (3)^{3(2-x)}$

$2 = 3(2-x)$

$2 = 6 - 3x$

$3x = 4$

$x = \frac{4}{3} = 1\frac{1}{3}$

10.4 $3^{2k-5} = 1$

$3^{2k-5} = 1^0$

$2k - 5 = 0$

$2k = 5$

$k = \frac{5}{2} = 2\frac{1}{2}$

11. Make the variable in the brackets the subject of the formula in the following:

11.1 $a = b^n$ (n)

$n = \log_b a$ Simple conversion to log form

11.2 $y = \log_{10} x$ (x)

$X = 10^y$

Simple conversion to an exponential form

11.3 $\log_2\left(\frac{a}{b}\right) = n$ (a)

$2^n = \frac{a}{b}$

$a = 2^n b$

11.4 $Q = Q_o e^{kt}$ (t)

$e^{kt} = \frac{Q}{Q_o}$

$\log_e e^{kt} = \log_e\left(\frac{Q}{Q_o}\right)$

$kt = \ln\left(\frac{Q}{Q_o}\right)$

$t = \frac{1}{k}\ln\left(\frac{Q}{Q_o}\right)$

Remember that: $\log_e = \ln$

12. The power gain of an amplifier, in decibels, is given by $G = 10\log_{10}\frac{P_o}{P_i}$, where P_i is the input power and P_o is the output power.

12.1 Make P_o the subject of the formula.

$\frac{G}{10} = \frac{10}{10}\log_{10}\left(\frac{P_o}{P_i}\right)$

$\frac{G}{10} = \log_{10}\left(\frac{P_o}{P_i}\right)$

$10^{\frac{G}{10}} = \frac{P_o}{P_i}$

$P_o = P_i \times 10^{\frac{G}{10}}$

12.2 Hence find the output power from an amplifier having a 45dB gain and an input signal power of 10 mW (answer correct to 2 significant figures)

$P_o = 0.01 \times 10^{\frac{45}{10}} =$

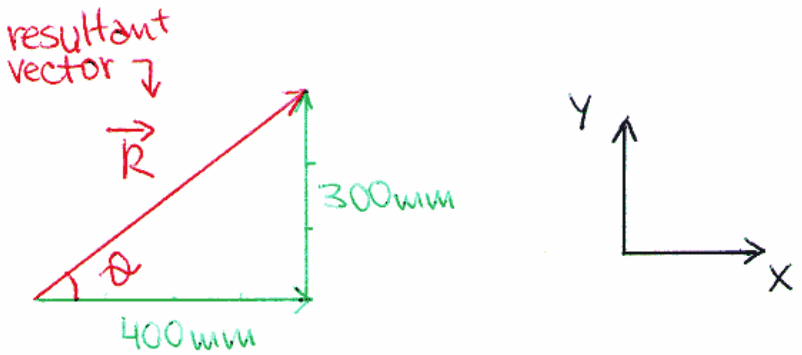
$= 0.01 \times 10^{4.5} =$

$= 0.01 \times 31623 =$

$= 316\ W =$

$= 320\ W$ (Rounding to 2 significant figures)

13. A robot arm needs to move 400 mm on the x-axis and 300 mm on the y-axis. Sketch these displacement vectors and the resultant displacement vector. Calculate the magnitude AND direction (angle) of the resultant displacement vector.



Magnitude of $\vec{R}$:

$R^2 = 400^2 + 300^2$

$R = \sqrt{400^2 + 300^2}$

$R = 500\ mm$

Angle of resultant displacement vector θ:

$\tan\theta = \frac{300}{400}$

$\theta = \tan^{-1}\left(\frac{300}{400}\right)$

$\theta = 36.9°$

------- **END OF TRIAL TEST/ASSIGNMENT** - ***Check your work!*** -------